Abstract—We investigate the performance of uncoded uplink transmissions in a broadband code division multiple access (CDMA) system using a tapped delay line (TDL) antenna array. First, the weights of a TDL antenna array are derived for broadband CDMA. Using these weights, we perform simulations to demonstrate how TDL antenna arrays can help reduce multi-access interference (MAI), allow the use of spreading codes with higher cross correlations, and increase the number of supportable users. Using the two-ray channel model, Kasami spreading codes, and placing users at the angles uniformly distributed around the antenna array, we identify the numbers of antennas and tap delays required to increase the number of supportable users from 15 in the baseline case to 30 and 60 with the uncoded BER fixed at 10^{-3}. At the signal-to-noise ratio (SNR) of 20 dB, one antenna with one tap is sufficient to support 15 users. For the same SNR, two antennas with four taps each can support 30 users. Finally, three antennas with six taps each can support 60 users.

Keywords—Antenna array, beamforming, code division multiple access.

I. INTRODUCTION

Compared to current mobile communication systems, next generation systems aim to provide higher data rates as well as to support more users. Broadband code division multiple access (CDMA) is a strong candidate technology for such systems. Two approaches of broadband CDMA exist: direct sequence CDMA (DS-CDMA) and multi-carrier CDMA (MC-CDMA). While MC-CDMA may be advantageous in terms of having less intersymbol interference (ISI), it was shown that DS-CDMA can achieve a comparable performance to MC-CDMA if proper frequency domain equalization is adopted [1,2]. We shall focus on DS-CDMA.

To support more users, nonorthogonal spreading codes with higher cross correlations may be used. However, the use of such codes results in higher multi-access interference (MAI). To reduce MAI, beamforming using an antenna array can be used. For narrowband systems, a beamformer can be implemented using gain and phase-shifting elements at each antenna. For broadband systems, the same structure is not sufficient since the channel is both space and frequency dependent. Consequently, we consider broadband beamforming using tapped delay lines (TDLs). Such a TDL beamformer acts as a spatial filter as well as a frequency-selective filter [3,4,5]. We shall investigate the performance of a minimum mean square error (MMSE) processor for a TDL beamformer in broadband CDMA systems.

In [6], the authors demonstrate the bit error rate (BER) improvement of broadband CDMA transmissions when a TDL beamformer with two antennas and five taps is used instead of an omni-directional antenna. In [7], the authors showed that an antenna array can be used in a narrowband CDMA system to increase the number of supportable users from 6 for one antenna to 63 for eight antennas at the uncoded BER equal to 10^{-3}. We shall extend their results by investigating how a TDL beamformer can help increase the number of supportable users in a broadband CDMA system. As the BER of 10^{-3} has been widely used as a benchmark for uncoded systems (e.g. [8,9]), we shall use 10^{-3} as the target BER.

To satisfy our objective, we perform the following tasks. First, we derive the weights of a TDL beamformer for a broadband CDMA system. These weights are different from the available expressions for narrowband systems, and can result in an improvement in BER.

Second, through simulations, we demonstrate how a TDL beamformer can help increase the number of supportable users in a broadband CDMA system. In particular, we quantify the number of antennas and tap delays required to increase the number of supportable users from 15 for one antenna and one tap to 30 and 60 respectively. In the investigation, the BER is fixed at 10^{-3}, while the signal-to-noise ratio (SNR) is fixed at 20 dB.

The rest of the paper proceeds as follows. Section II contains the system model for broadband CDMA transmissions and the TDL beamformer structure. Section III contains the derivation of the optimal weights of a TDL beamformer according to the MMSE criterion. Section IV provides simulation results that demonstrate how a TDL beamformer can help increase the number of supportable users. Finally, section V summarizes our work.

II. SYSTEM MODEL

A. Transmitted Signals

Assume that there are $K$ users in a broadband CDMA system. Let $G$ be the processing gain or equivalently the spreading factor of CDMA. Let $e_k^k = (c_0^k, ..., c_{G-1}^k)$ be the CDMA code for user $k$, $k \in \{1, ..., K\}$. Assume that binary phase shift keying (BPSK) is used as a modulation technique. Denote the $i$th symbol transmitted from user $k$ by $a_i^k$, $i \in \{0,1,...\}$. Assume that $a_i^k \in \{1,-1\}$. The baseband pulse for BPSK is denoted by $p(t)$. Let $T_b$ and $T_c$ denote the bit and chip periods respectively. The combined baseband CDMA signal from all users is
\[ m_y(t) = \sum_{k=1}^{K} \sum_{l=0}^{G-1} \sum_{j=0}^{G-1} a'[l] k_i^* p(t - gT_c - iT_i). \]  \hspace{1cm} (1)

The corresponding passband signal with carrier frequency \( f_c \) is written as

\[ m_y(t) = \sqrt{2} m_y(t) \cos(2\pi f_c t). \]  \hspace{1cm} (2)

B. Two-Ray Fading Channel Model

Radio transmissions through realistic communication channels suffer the adverse effects of multipath fading. We shall adopt the two-ray channel model to capture the multipath fading effects. Fig. 1 illustrates the two-ray channel model. We assume that the channel is slow fading and can be described using the following parameters. Let \( \gamma_1 \) and \( \gamma_2 \) denote the gains or losses on the direct and reflected paths. Let \( \theta_1 \) and \( \theta_2 \) denote the arrival angles, \( \tau_1 \) and \( \tau_2 \) denote the weight for the relative propagation delay of the two paths with respect to the receive antenna array. Finally, let \( r_{12} \) denote the relative propagation delay of the two paths at the first antenna in the array.

C. Receive Signals

Consider a receiver with a linear array of \( L \) antennas with antenna spacing \( d \). Let \( x_l(t) \) be the receive signal at antenna \( l, l \in \{1, \ldots, L\} \). Assume there is an additive white Gaussian noise (AWGN) process \( n_l(t) \) at antenna \( l \). The receive combined signal with the assumption of line-of-sight delay to antenna 1 being zero can be expressed as

\[ x_l(t) = \sqrt{2} \gamma_l m_y(t) \cos(2\pi f_c (t - (l-1)\tau_1)) + \sqrt{2} \gamma_l m_y(t) \cos(2\pi f_c (t - (l-1)\tau_2)) + n_l(t), \]  \hspace{1cm} (3)

where we define \( \tau_1 = d \sin \theta_1 / c \) and \( \tau_2 = d \sin \theta_2 / c \) as the time lags between adjacent antennas for the direct and reflected paths respectively. Note that \( c \) is equal to the speed of light.

C. TDL Beamformer

In a TDL beamformer, there are \( L \) antennas each of which is equipped with \( J \) taps separated by delay \( T_D \). Let \( w_{jl}, l \in \{1, \ldots, L\}, j \in \{1, \ldots, J\} \), denote the weight for the signal on antenna \( l \) at tap \( j \). (Let * denote the complex conjugate of \( x \). In addition, let \( x^T \) and \( x^\dagger \) denote the transpose and conjugate transpose of \( x \).)

The first tap output corresponds to the receive signal without delay, while the \( j \)th tap output corresponds to the signal delayed by \((j-1)T_D\). The output \( y(t) \) of a TDL beamformer is

\[ y(t) = \sum_{j=1}^{J} \sum_{l=1}^{L} \gamma_j \gamma_l m_y(t) w_{jl}^*, \]  \hspace{1cm} (4)

For convenience, define vectors \( \mathbf{w} \) and \( \mathbf{x}(t) \) as

\[ \mathbf{w} = [w_1^*, \ldots, w_L^*, w_1, \ldots, w_J]^T, \]  \hspace{1cm} (5)

\[ \mathbf{x}(t) = [x_1(t), \ldots, x_L(t)]^T, \]  \hspace{1cm} (6)

where \( x_j(t) = x_j(t), \ldots, x_j(t-(J-1)T_D)]^T \).

From the definitions in (5) and (6), the beamformer output \( y(t) \) in (4) can be expressed concisely as

\[ y(t) = \mathbf{w}^T \mathbf{x}(t). \]  \hspace{1cm} (7)

The optimal tap weights \( \mathbf{w} \) of a TDL beamformer, denoted by \( \mathbf{w}_{opt} \), can be found using the Wiener Hop equations described in [10] to be

\[ \mathbf{w}_{opt} = \mathbf{R}^{-1} \mathbf{p}, \]  \hspace{1cm} (8)

where \( \mathbf{R} = \mathbb{E}[\mathbf{X}(t)\mathbf{X}(t)^T] \) is the \( L \times L \) covariance matrix of the received signal vector \( \mathbf{X}(t) \), and \( \mathbf{p} = \mathbb{E}[\mathbf{X}(t)d^*(t)] \) is the \( L \times 1 \) correlation vector of \( \mathbf{X}(t) \) and the reference signal \( d(t) \). In computing \( \mathbf{w}_{opt} \) in (8), we assume the knowledge of the desired user location.

D. Signal Detection with a TDL Beamformer

We consider signal detection in a CDMA system with a TDL beamformer. In particular, the beamformer output is first demodulated using a local oscillator operating at the carrier frequency \( f_c \). The demodulated output is then despread using the CDMA code for the desired user. The result is then passed through a matched filter whose output samples taken at every bit period \( T_b \) enter the threshold decision device.

III. DERIVATION OF OPTIMAL TDL WEIGHTS FOR BROADBAND CDMA SYSTEMS

In this section, we derive an analytical expression for the optimal weight \( \mathbf{w}_{opt} \) for a TDL beamformer to be used in a broadband CDMA system. First, we write the correlation function of the signals received at two different antenna-tap values \((i, j)\) and \((i', j')\) as

\[ \rho_{ij}(\Delta) = \mathbb{E}[X_i(t - (j-1)T_D)X_j(t - (j' - 1)T_D)] \]  \hspace{1cm} (9)

where \( \Delta = (j - j')T_D \),

\[ \Delta_1 = (l-1)\tau_1 + jT_D - (l'-1)\tau_1 - j'T_D, \]  \hspace{1cm} (9)

\[ \Delta_2 = (l-1)\tau_2 + jT_D - (l'-1)\tau_1 - j'T_D, \]  \hspace{1cm} (9)
\[
\Delta_1 = (l-1)r_z + jT_D - (l'-1)r_z - \tau_2 - j' T_D ,
\]
\[
\Delta_4 = (l-1)r_z + jT_D - (l'-1)r_z - j' T_D .
\]

The autocorrelation function of passband signal \( \varphi_{M_{M_a}}(\cdot) \) can be described in terms of the autocorrelation of the baseband signal \( \varphi_{m_{M_a}}(\cdot) \) as follows.

\[
\varphi_{M_{M_a}}(\Delta) = \mathbb{E} \left[ M_p(t) M_p(t+\Delta) \right] \\
= \mathbb{E} \left[ M_{M_a}(t) M_{M_a}(t+\Delta) \right] \cos(2\pi f_0 \Delta) \\
= \varphi_{m_{M_a}}(\Delta) \cos(2\pi f_0 \Delta)
\]

Note that, in narrowband systems, we can use \( m_b(t) \approx m_b(t+\Delta) \), yielding \( \varphi_{m_{M_a}}(\Delta) = \mathbb{E} [M_{M_a}(t) M_{M_a}(t+\Delta)] \approx \mathbb{E} [M_{M_a}^2(t)] \), which says that the \( \varphi_{m_{M_a}}(\Delta) \) is approximately equal to the baseband signal power. Since the approximation \( m_b(t) \approx m_b(t+\Delta) \) is not valid for broadband systems, we proceed to derive \( \varphi_{M_{M_a}}(\Delta) \).

For statistically independent transmitted symbols, we have the following fact.

\[
\mathbb{E} \left[ A^* A^* | i, i' \right] = \begin{cases} 1, & k = k', i = i', \\ 0, & \text{otherwise} \end{cases} (12)
\]

We now make an additional assumption that different chip values of CDMA codes are statistically independent. This assumption allows us to compute the approximated expression for \( \varphi_{m_{M_a}}(\Delta) \). With the assumption, we can extend the property in (12) to the following statement.

\[
\mathbb{E} \left[ c_g A^* A^* | i, i' \right] = \begin{cases} 1, & k = k', i = i', \\ 0, & \text{otherwise} \end{cases} (13)
\]

From (1) and (13), we can write \( \varphi_{m_{M_a}}(\Delta) \) as

\[
\varphi_{M_{M_a}}(\Delta) = \mathbb{E} [M_{M_a}(t) M_{M_a}(t+\Delta)] \\
= \mathbb{E} \left[ \sum_{i,j,l=1}^{G-1} \sum_{g=0}^{G-1} c_g A^* A^* \right] \mathbb{E} p(t-g T_c - iT_p) p(t-g T_c - iT_p + \Delta) \\
= K \sum_{i, j=1}^{G-1} p(t-g T_c - iT_p) p(t-g T_c - iT_p + \Delta) \\
\]

Define \( \varphi(t, \Delta) \) illustrated in Fig. 2 as

\[
\varphi(t, \Delta) = \sum_{i=1}^{G-1} \sum_{g=0}^{G-1} p(t-g T_c - iT_p) p(t-g T_c - iT_p + \Delta) . \quad (15)
\]

Note that \( \varphi(t, \Delta) \) is periodic with period \( T \) and \( \varphi(t, \Delta) = 0 \) if \( \Delta > T \). Since \( \varphi(t, \Delta) \) is time-dependent, for the TDL weight computation, we can use the time average value of \( \varphi(t, \Delta) \). This averaging is similar to the process of computing autocorrelation of cyclostationary processes [11]. The time average value of \( \varphi(t, \Delta) \) over a single period is computed as follows.

\[
\bar{\varphi}(\Delta) = \frac{1}{T_{c}} \int_{T_c}^{T_c} \varphi(t, \Delta) dt = \begin{cases} \frac{1}{T_c} \int_{0}^{\Delta} \frac{1}{T_c}, & \Delta \leq T_c \\
0, & \text{otherwise} \end{cases} (16)
\]

By setting \( \varphi(t, \Delta) = \bar{\varphi}(\Delta) \) and using (9), (10), (11), (14), (15), and (16), we can compute \( \varphi_{M_a}(\lambda) \) which is used to compute the optimal TDL weights in (8).

### III. RESULTS AND DISCUSSIONS

To investigate the performance of a TDL beamformer for broadband CDMA, we conduct simulations using MATLAB software. In the simulation model, the carrier frequency is set to 2 GHz. The transmission bandwidth is 100 MHz. (These frequency and bandwidth values are based on the UMTS-2000 standard.) The data rate is 400 kbps. The CDMA processing gain is 255. The required BER is set at 10^{-3}. Small and large Kasami sequences are used as CDMA codes. The two-ray channel model has direct and reflected path losses of 0.8 and 0.6 respectively. Users are located at the angles uniformly distributed around the antenna array. The angles of reflected paths are randomly generated according to the uniform distribution. For a TDL beamformer, the antenna separation is set at half the carrier wavelength, while the tap delay is set to correspond to the propagation distance of a quarter of the carrier wavelength. These choices are typical for TDL beamforming [3].

We validated the simulations with the work in [6]. In addition, we also validated the simulations with the BER estimate using the Q function. Details of validation are available in [12] and are omitted here.

#### A. Performance Comparison Using Broadband and Narrowband Optimal Weight Expressions

To demonstrate the advantages of using the optimal TDL weights derived especially for broadband CDMA in section III, we present the simulation results for 2 antennas and 4 taps in Fig. 3. Note that the broadband expressions for the optimal weights yield a better performance for sufficiently high signal-to-noise ratio (SNR). In addition, they also give a lower BER floor. Consequently, for the rest of the simulations, we use the broadband expressions for the optimal TDL weights.
To provide for the propagation distance of 25 and processing gain 255 can support only 16 users, we use a case, i.e., 20-dB SNR and 10 port 30 users at the same SNR and BER as in the baseline nas and taps. Fig. 5 shows the simulation results that include −

B. TDL Beamformer with Single Antenna and Single Tap

Motivated by the fact that small Kasami sequences with processing gain 255 can support only 16 users, we use a case, i.e., 20-dB SNR and 10

C. Supporting 30 and 60 Users

To increase the number of supportable users from 15 to 30, we perform simulations with higher number of antennas and taps. Fig. 5 shows the simulation results that include the beamformer capable of supporting 30 users. We observe that 2 antennas and 4 taps are sufficient to support 30 users at the same SNR and BER as in the baseline case, i.e., 20-dB SNR and 10^−2-BER.

Note that 2 antennas and 6 taps cannot help improve the performance significantly when compared to 2 antennas and 4 taps. We observe that a good rule of thumb is to set the number of taps equal to twice the number of antennas, i.e., setting J = 2L. This rule can be explained as follows. We have the antenna separation of λ/2 and the tap delay corresponding to the propagation distance of λ/4, where λ is the carrier wavelength. The receive signals at two different antennas in a linear array with L antennas cannot have a path length difference exceeding Lλ/2. To provide for the propagation distance of Lλ/2, (Lλ/2)/(λ/4) = 2L tap delays are sufficient.

Fig. 5 also shows the simulation results that include the beamformer capable of supporting 60 users. We observe that 3 antennas and 6 taps are sufficient to support 60 users at the SNR of 20 dB and the BER below 10^−3.

IV. CONCLUSIONS

We investigated the performance of a TDL beamformer in broadband CDMA systems. We first derived the expressions for the optimal TDL weights for broadband CDMA, and demonstrated their superior performance compared to the already available narrowband expressions. In addition, we demonstrated how TDL beamforming can be used to reduce MAI interference, allow the use of spreading codes with higher cross correlations, and increase the number of supportable users. At the SNR of 20 dB and the BER requirement of 10^−3, 1 antenna with 1 tap is sufficient to support 15 users. For the same SNR and BER requirement, 2 antennas with 4 taps each can support 30 users; 3 antennas with 6 taps each can support 60 users.

REFERENCES